

# 一类中立型 Cohen-Grossberg 神经网络的概周期解

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[摘要] 研究一类具有混合时滞的中立型 Cohen-Grossberg 神经网络。通过建立线性辅助方程, 得到该神经网络存在唯一的概周期解的新结果, 同时也给出此概周期解的存在范围。

[关键词] Cohen-Grossberg 神经网络; 中立型; 概周期解

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## The Almost Periodic Solution for a Class of Neutral Cohen-Grossberg Neural Networks

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**Abstract:** A class of neutral Cohen-Grossberg neural networks with mixed time delays was studied. By establishing a linear auxiliary equation, a new result that the neural network had a unique almost periodic solution was obtained. At the same time, the existence range of this almost periodic solution was given.

**Keywords:** Cohen-Grossberg neural networks; neutral; almost periodic solution

## 0 引言

Cohen-Grossberg 神经网络在模式识别、并行计算、优化、信号和图像处理等不同领域都有广泛的应用, 许多学者研究了各种具有时滞的 Cohen-Grossberg 神经网络的平衡点、周期解、概周期解的存在性、唯一性、稳定性等, 并取得了一些很好的研究成果<sup>[1-4]</sup>。但是, 由于神经细胞在现实世界中具有复杂的动态特性, 为了更准确反映神经元反应过程的特性, 有必要在神经系统的数学模型中加入一些关于过去状态的导数的信息, 这种新型的神经网络被称为中立型神经网络。近年来, 一些学者主要利用 Lyapunov 泛函、线性矩阵不等式、积分不等式、重合度理论、M 矩阵等方法, 研究了各种中立型 Cohen-Grossberg 神经网络的动力学行为, 并得到了一些新的结论<sup>[5-6]</sup>。然而, 从现有文献来看, 对于具有时滞的中立型 Cohen-Grossberg 神经网络的概周期解的相关问题的研究还是比较少。基于此, 本文研究如下一类具有混合时滞的中立型 Cohen-Grossberg 神经网络

$$\begin{aligned} x_i'(t) = & -\alpha_i(t, x_i(t)) \left[ \beta_i(t, x_i(t)) - \sum_{j=1}^n (a_{ij}(t) f_j(x_j(t))) - \sum_{j=1}^n (b_{ij}(t) g_j(x_j(t - \tau_{ij}(t)))) - \right. \\ & \left. \sum_{j=1}^n (c_{ij}(t) \int_0^{+\infty} k_{ij}(s) h_j(x_j'(t-s)) ds) + I_i(t) \right], i = 1, 2, \dots, n \end{aligned} \quad (1)$$

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的概周期解。其中:  $x_i(t)$  表示第  $i$  个神经元在  $t$  时刻的状态;  $\alpha_i(t, \cdot)$  表示放大函数;  $\beta_i(t, \cdot)$  表示行为函数;  $a_{ij}(t), b_{ij}(t), c_{ij}(t)$  表示神经元之间的连接权重;  $f_j(\cdot), g_j(\cdot), h_j(\cdot)$  表示神经元激活函数;  $\tau_{ij}(t)$  表示传输时滞且满足  $0 \leq \tau_{ij}(t) \leq \tau$  ( $\tau > 0$  为常数);  $k_{ij}(s)$  表示分布时滞核函数;  $I_i(t)$  表示第  $i$  个神经元在  $t$  时刻的外部输入。本文通过建立线性辅助方程的技巧, 得到了系统 (1) 存在唯一的概周期解的新结果, 同时也给出了此概周期解的存在范围。

## 1 主要结果

对于系统 (1), 作如下假设:  $H_1)$   $\alpha_i(t, x), \beta_i(t, x)$  对  $x \in \mathbf{R}$  关于  $t$  是一致概周期函数,  $a_{ij}(t), b_{ij}(t), c_{ij}(t), \tau_{ij}(t), I_i(t)$  都是概周期函数;  $H_2)$  时滞核函数  $k_{ij}: [0, +\infty) \rightarrow \mathbf{R}$  连续可积, 且存在正数  $K_{ij}$ , 使得  $\int_0^{+\infty} |k_{ij}(s)| ds \leq K_{ij}$ ;  $H_3)$  存在常数  $\bar{\alpha}_i > 0, L_i > 0$ , 使得  $0 < \alpha_i(t, x) \leq \bar{\alpha}_i, |\alpha_i(t, x) - \alpha_i(t, y)| \leq L_i |x - y|, \forall t, x, y \in \mathbf{R}$ ;  $H_4)$  存在正数  $\omega_i, \mu_i, Y_i$ , 使得  $0 < \omega_i \leq [\alpha_i(t, x)\beta_i(t, x) - \alpha_i(t, y)\beta_i(t, y)]/(x - y) \leq \mu_i, |\alpha_i(t, 0)\beta_i(t, 0)| \leq Y_i, \forall t, x, y \in \mathbf{R}, x \neq y$ ;  $H_5)$  存在函数  $f_{1j}, f_{2j}, g_{1j}, g_{2j}, h_{1j}, h_{2j}$  及正数  $l'_{1j}, l'_{2j}, l''_{1j}, l''_{2j}, l^h_{1j}, l^h_{2j}, M'_j, M''_j, M^h_j$ , 使得对任意的  $x, y \in \mathbf{R}$ , 有:  $f_j(x) = f_{1j}(x)f_{2j}(x), |f_{1j}(x) - f_{1j}(y)| \leq l'_{1j}|x - y|, |f_{2j}(x) - f_{2j}(y)| \leq l'_{2j}|x - y|, f_{1j}(0) = 0, |f_{2j}(x)| \leq M'_j, g_j(x) = g_{1j}(x)g_{2j}(x), |g_{1j}(x) - g_{1j}(y)| \leq l''_{1j}|x - y|, |g_{2j}(x) - g_{2j}(y)| \leq l''_{2j}|x - y|, g_{1j}(0) = 0, |g_{2j}(x)| \leq M''_j, h_j(x) = h_{1j}(x)h_{2j}(x), |h_{1j}(x) - h_{1j}(y)| \leq l^h_{1j}|x - y|, |h_{2j}(x) - h_{2j}(y)| \leq l^h_{2j}|x - y|, h_{1j}(0) = 0, |h_{2j}(x)| \leq M^h_j$ ;  $H_6)$   $\lambda = \max\{\max_{1 \leq i \leq n}(2\lambda_i/(\mu_i + \omega_i)), \max_{1 \leq i \leq n}(2\lambda_i)\} < 1$ , 其中:  $\lambda_i = (\mu_i - \omega_i)/2 + \bar{\alpha}_i \sum_{j=1}^n [\bar{a}_{ij}l'_{1j}(M'_j + (l'_{2j}\zeta)/(1 - \eta) + \bar{b}_{ij}l''_{1j}(M''_j + (l''_{2j}\zeta)/(1 - \eta)) + \bar{c}_{ij}l^h_{1j}K_{ij}(M^h_j + (l^h_{2j}\zeta)/(1 - \eta))] + L_i[\zeta/(1 - \eta) \sum_{j=1}^n (\bar{a}_{ij}l'_{1j}M'_j + \bar{b}_{ij}l''_{1j}M''_j + \bar{c}_{ij}l^h_{1j}M^h_j K_{ij}) + \bar{I}_i]$ ,  $\bar{a}_{ij} = \sup_{t \in \mathbf{R}}\{|a_{ij}(t)|\}, \bar{b}_{ij} = \sup_{t \in \mathbf{R}}\{|b_{ij}(t)|\}, \bar{c}_{ij} = \sup_{t \in \mathbf{R}}\{|c_{ij}(t)|\}, \bar{I}_i = \sup_{t \in \mathbf{R}}\{|I_i(t)|\}, \eta = \max\{\max_{1 \leq i \leq n}[(2\eta_i)/(\mu_i + \omega_i)], \max_{1 \leq i \leq n}(2\eta_i)\}, \zeta = \max\{\max_{1 \leq i \leq n}[(2\zeta_i)/(\mu_i + \omega_i)], \max_{1 \leq i \leq n}(2\zeta_i)\}, \eta_i = (\mu_i - \omega_i)/2 + \bar{\alpha}_i \sum_{j=1}^n (\bar{a}_{ij}l'_{1j}M'_j + \bar{b}_{ij}l''_{1j}M''_j + \bar{c}_{ij}K_{ij}l^h_{1j}M^h_j) + L_i\bar{I}_i, \zeta_i = Y_i + \bar{\alpha}_i\bar{I}_i$ 。

设  $\Omega = \{\varphi(t) | \varphi(t) = (\varphi_1(t), \varphi_2(t), \dots, \varphi_n(t))^T; \mathbf{R} \rightarrow \mathbf{R}^n \text{ 为概周期函数且 } \varphi'(t) \text{ 也是概周期函数}\}$ , 对任意的  $\varphi(t) \in \Omega$ , 定义范数  $\|\varphi\| = \max\{\sup_{t \in \mathbf{R}}[\max_{1 \leq i \leq n}|\varphi_i(t)|], \sup_{t \in \mathbf{R}}[\max_{1 \leq i \leq n}|\varphi_i'(t)|]\}$ , 那么在此范数下  $\Omega$  是一个 Banach 空间。

**定理 1** 假设条件  $H_1) - H_6)$  成立, 则系统 (1) 在  $\Omega^* = \{\varphi | \varphi \in \Omega, \|\varphi\| \leq \zeta/(1 - \eta)\}$  内存在唯一的概周期解。

## 2 定理 1 的证明

对任意的  $\varphi(t) \in \Omega$ , 由条件  $H_1)$ 、 $H_2)$  可构造如下线性概周期微分系统

$$x_i'(t) = -(\mu_i + \omega_i)x_i(t)/2 + (\mu_i + \omega_i)\varphi_i(t)/2 - \alpha_i(t, \varphi_i(t))[\beta_i(t, \varphi_i(t)) - \sum_{j=1}^n (a_{ij}(t)f_j(\varphi_j(t))) - \sum_{j=1}^n (b_{ij}(t)g_j(\varphi_j(t - \tau_{ij}(t)))) - \sum_{j=1}^n (c_{ij}(t) \int_0^{+\infty} k_{ij}(s)h_j(\varphi_j'(t - s))ds) + I_i(t)], i = 1, 2, \dots, n. \quad (2)$$

因为  $-(\mu_i + \omega_i)/2 < 0$ , 所以根据文献 [7] 的定理 7.7 可知, 系统 (2) 存在唯一的概周期解  $x^\varphi(t) = (x_1^\varphi(t), x_2^\varphi(t), \dots, x_n^\varphi(t))^T$ , 其中,

$$x_i^\varphi(t) = \int_{-\infty}^t \exp(-(\mu_i + \omega_i)(t - s)/2) \{(\mu_i + \omega_i)\varphi_i(s)/2 - \alpha_i(s, \varphi_i(s))[\beta_i(s, \varphi_i(s)) - \sum_{j=1}^n a_{ij}(s)f_j(\varphi_j(s)) -$$

$$\sum_{j=1}^n b_{ij}(s)g_j(\varphi_j(s-\tau_{ij}(s))) - \sum_{j=1}^n c_{ij}(s)\int_0^{+\infty} k_{ij}(u)h_j(\varphi'_j(s-u))du + I_i(s) \} ds. \quad (3)$$

因为  $x^\varphi(t) \in \Omega$ , 所以可定义映射  $\Phi: \Omega^* \rightarrow \Omega$  为  $\Phi(\varphi) = x^\varphi$ . 对任意的  $\varphi \in \Omega^*$ , 下面证明  $x^\varphi(t) \in \Omega^*$ .

由式 (2)、式 (3) 及条件  $H_2) \sim H_6)$  可得,  $|x_i^\varphi(t)| = |\int_{-\infty}^t \exp(-(\mu_i + \omega_i)(t-s)/2) \{ (\mu_i + \omega_i)\varphi_i(s)/2 - \alpha_i(s, \varphi_i(s))\beta_i(s, \varphi_i(s)) + \alpha_i(s, 0)\beta_i(s, 0) - \alpha_i(s, 0)\beta_i(s, 0) + \alpha_i(s, \varphi_i(s))$   
 $[\sum_{j=1}^n a_{ij}(s)f_{ij}(\varphi_j(s))f_{2j}(\varphi_j(s)) + \sum_{j=1}^n b_{ij}(s)g_{ij}(\varphi_j(s-\tau_{ij}(s)))g_{2j}(\varphi_j(s-\tau_{ij}(s))) + \sum_{j=1}^n c_{ij}(s)$   
 $\int_0^{+\infty} k_{ij}(u)h_{ij}(\varphi'_j(s-u))h_{2j}(\varphi'_j(s-u))du] - \alpha_i(s, \varphi_i(s))I_i(s) + \alpha_i(s, 0)I_i(s) - \alpha_i(s, 0)I_i(s) \} ds| \leq$   
 $|\int_{-\infty}^t \exp(-(\mu_i + \omega_i)(t-s)/2) \{ [(\mu_i - \omega_i)/2 + \bar{\alpha}_i \sum_{j=1}^n (\bar{a}_{ij}l_{ij}^f M_j^f + \bar{b}_{ij}l_{ij}^g M_j^g + \bar{c}_{ij}K_{ij}l_{ij}^h M_j^h) + L_i \bar{I}_i] \|\varphi\| + Y_i +$   
 $\bar{\alpha}_i \bar{I}_i \} ds| = 2\|\varphi\|/(\mu_i + \omega_i) [(\mu_i - \omega_i)/2 + \bar{\alpha}_i \sum_{j=1}^n (\bar{a}_{ij}l_{ij}^f M_j^f + \bar{b}_{ij}l_{ij}^g M_j^g + \bar{c}_{ij}K_{ij}l_{ij}^h M_j^h) + L_i \bar{I}_i] + 2(Y_i + \bar{\alpha}_i \bar{I}_i)/(\mu_i + \omega_i)$   
 $= 2\eta_i \|\varphi\|/(\mu_i + \omega_i) + 2\zeta_i/(\mu_i + \omega_i) \leq \eta \|\varphi\| + \zeta, |(x_i^\varphi(t))'| = |-(\mu_i + \omega_i)x_i^\varphi(t)/2 + (\mu_i + \omega_i)\varphi_i(t)/2 - \alpha_i(t, \varphi_i(t))[\beta_i(t, \varphi_i(t)) - \sum_{j=1}^n a_{ij}(t)f_j(\varphi_j(t)) - \sum_{j=1}^n (b_{ij}(t)g_j(\varphi_j(t-\tau_{ij}(t)))) -$   
 $\sum_{j=1}^n (c_{ij}(t)\int_0^{+\infty} k_{ij}(s)h_j(\varphi'_j(t-s))ds) + I_i(t)]| \leq (\mu_i + \omega_i)|x_i^\varphi(t)|/2 + |(\mu_i + \omega_i)\varphi_i(t)/2 - \alpha_i(t,$   
 $\varphi_i(t))[\beta_i(t, \varphi_i(t)) - \sum_{j=1}^n a_{ij}(t)f_j(\varphi_j(t)) - \sum_{j=1}^n b_{ij}(t)g_j(\varphi_j(t-\tau_{ij}(t))) - \sum_{j=1}^n (c_{ij}(t)\int_0^{+\infty} k_{ij}(s)h_j(\varphi'_j(t-s))ds) + I_i(t)]| \leq$   
 $((\mu_i + \omega_i)/2)(2\eta_i \|\varphi\|/(\mu_i + \omega_i) + 2\zeta_i/(\mu_i + \omega_i)) + \eta_i \|\varphi\| + \zeta_i = 2\eta_i \|\varphi\| + 2\zeta_i \leq \eta \|\varphi\| + \zeta.$  故  $\|x^\varphi\| = \max_{t \in \mathbf{R}} \{ \sup_{1 \leq i \leq n} |x_i^\varphi(t)|, \sup_{t \in \mathbf{R}} \{ \max_{1 \leq i \leq n} |(x_i^\varphi(t))'| \} \} \leq \eta \|\varphi\| + \zeta \leq \zeta\eta/(1-\eta) + \zeta = \zeta/(1-\eta)$ , 即  $x^\varphi(t) \in \Omega^*$ . 所以对任意的  $\varphi \in \Omega^*$ , 都有  $\Phi(\varphi) \in \Omega^*$ , 即映射  $\Phi$  是  $\Omega^*$  到  $\Omega^*$  的自映射. 下面证明  $\Phi: \Omega^* \rightarrow \Omega^*$  是一个压缩映射.

对于任意的  $\varphi, \phi \in \Omega^*$ , 有:  $|x_i^\varphi(t) - x_i^\phi(t)| = |\int_{-\infty}^t \exp(-(\mu_i + \omega_i)(t-s)/2) \{ (\mu_i + \omega_i)(\varphi_i(s) - \phi_i(s))/2 - \alpha_i(s, \varphi_i(s))\beta_i(s, \varphi_i(s)) + \alpha_i(s, \phi_i(s))\beta_i(s, \phi_i(s)) + \alpha_i(s, \varphi_i(s))$   
 $[ \sum_{j=1}^n (a_{ij}(s)(f_j(\varphi_j(s)) - f_j(\phi_j(s)))) + \sum_{j=1}^n (b_{ij}(s)(g_j(\varphi_j(s-\tau_{ij}(s))) - g_j(\phi_j(s-\tau_{ij}(s)))) + \sum_{j=1}^n (c_{ij}(s)\int_0^{+\infty} k_{ij}(u)(h_j(\varphi'_j(s-u)) - h_j(\phi'_j(s-u)))du] + (\alpha_i(s, \varphi_i(s)) - \alpha_i(s, \phi_i(s)))$   
 $[ \sum_{j=1}^n (a_{ij}(s)f_j(\phi_j(s))) + \sum_{j=1}^n (b_{ij}(s)g_j(\phi_j(s-\tau_{ij}(s)))) + \sum_{j=1}^n (c_{ij}(s)\int_0^{+\infty} k_{ij}(u)h_j(\phi'_j(s-u))du) - I_i(s) \} ds| \leq \int_{-\infty}^t e^{-(\mu_i + \omega_i)(t-s)/2} \{ (\mu_i - \omega_i)|\varphi_i(s) - \phi_i(s)|/2 +$   
 $\bar{\alpha}_i [ \sum_{j=1}^n (\bar{a}_{ij}(|f_{1j}(\varphi_j(s)) - f_{1j}(\phi_j(s))| |f_{2j}(\varphi_j(s))| + |f_{1j}(\phi_j(s))| |f_{2j}(\varphi_j(s)) - f_{2j}(\phi_j(s))|)) + \sum_{j=1}^n (\bar{b}_{ij}(|g_{1j}(\varphi_j(s-\tau_{ij}(s))) - g_{1j}(\phi_j(s-\tau_{ij}(s)))| |g_{2j}(\varphi_j(s-\tau_{ij}(s)))| +$   
 $|g_{1j}(\phi_j(s-\tau_{ij}(s)))| |g_{2j}(\varphi_j(s-\tau_{ij}(s))) - g_{2j}(\phi_j(s-\tau_{ij}(s)))|)) + \sum_{j=1}^n (\bar{c}_{ij}\int_0^{+\infty} |k_{ij}(u)|$   
 $(|h_{1j}(\varphi'_j(s-u)) - h_{1j}(\phi'_j(s-u))| |h_{2j}(\varphi'_j(s-u))| + |h_{1j}(\phi'_j(s-u))| |h_{2j}(\varphi'_j(s-u)) - h_{2j}(\phi'_j(s-u))|)du] + L_i|\varphi_i(s) - \phi_i(s)| [ \sum_{j=1}^n (\bar{a}_{ij}l_{ij}^f M_j^f |\phi_j(s)|) + \sum_{j=1}^n (\bar{b}_{ij}l_{ij}^g M_j^g |\phi_j(s-\tau_{ij}(s))|) +$

$$\sum_{j=1}^n (\bar{c}_{ij} \int_0^{+\infty} |k_{ij}(u)| l_{ij}^h M_j^h |\phi'_j(s-u)| du + \bar{I}_i] ds \leq \int_{-\infty}^t \|\varphi - \phi\| \exp(-(\mu_i + \omega_i)(t-s)/2) \{(\mu_i - \omega_i)/2 +$$

$$\bar{\alpha}_i \sum_{j=1}^n [\bar{a}_{ij} l_{ij}^f (M_j^f + l_{2j}^f \zeta/(1-\eta)) + \bar{b}_{ij} l_{ij}^g (M_j^g + l_{2j}^g \zeta/(1-\eta)) + \bar{c}_{ij} l_{ij}^h K_{ij} (M_j^h + l_{2j}^h \zeta/(1-\eta))] + L_i [(\zeta/(1-\eta))$$

$$\eta) \sum_{j=1}^n (\bar{a}_{ij} l_{ij}^f M_j^f + \bar{b}_{ij} l_{ij}^g M_j^g + \bar{c}_{ij} l_{ij}^h M_j^h K_{ij}) + \bar{I}_i] \} ds \leq (2/(\mu_i + \omega_i)) \{(\mu_i - \omega_i)/2 + \bar{\alpha}_i \sum_{j=1}^n [\bar{a}_{ij} l_{ij}^f (M_j^f + l_{2j}^f \zeta/(1-\eta)) + \bar{b}_{ij} l_{ij}^g (M_j^g + l_{2j}^g \zeta/(1-\eta)) + \bar{c}_{ij} l_{ij}^h K_{ij} (M_j^h + l_{2j}^h \zeta/(1-\eta))] + L_i [(\zeta/(1-\eta)) \sum_{j=1}^n (\bar{a}_{ij} l_{ij}^f M_j^f + \bar{b}_{ij} l_{ij}^g M_j^g + \bar{c}_{ij} l_{ij}^h M_j^h K_{ij}) + \bar{I}_i] \} \|\varphi - \phi\| = 2\lambda_i \|\varphi - \phi\|/(\mu_i + \omega_i), \quad |(x_i^\varphi(t))' - (x_i^\phi(t))'| = |-(\mu_i + \omega_i)(x_i^\varphi(t) - x_i^\phi(t))/2 + (\mu_i + \omega_i)(\varphi_i(t) - \phi_i(t))/2 - \alpha_i(t, \varphi_i(t))\beta_i(t, \varphi_i(t)) + \alpha_i(t, \phi_i(t))\beta_i(t, \phi_i(t)) + \alpha_i(t, \varphi_i(t))[\sum_{j=1}^n (a_{ij}(t)(f_j(\varphi_j(t)) - f_j(\phi_j(t)))) + \sum_{j=1}^n (b_{ij}(t)(g_j(\varphi_j(t - \tau_{ij}(t))) - g_j(\phi_j(t - \tau_{ij}(t)))) + \sum_{j=1}^n (c_{ij}(t) \int_0^{+\infty} k_{ij}(u)(h_j(\varphi'_j(t-u)) - h_j(\phi'_j(t-u))) du)] + (\alpha_i(t, \varphi_i(t)) - \alpha_i(t, \phi_i(t)))[\sum_{j=1}^n (a_{ij}(t)f_j(\phi_j(t))) + \sum_{j=1}^n (b_{ij}(t)g_j(\phi_j(t - \tau_{ij}(t)))) + \sum_{j=1}^n (c_{ij}(t) \int_0^{+\infty} k_{ij}(u)h_j(\phi'_j(t-u)) du) - I_i(t)] \leq 2\|\varphi - \phi\| \{(\mu_i - \omega_i)/2 + \bar{\alpha}_i \sum_{j=1}^n [\bar{a}_{ij} l_{ij}^f (M_j^f + l_{2j}^f \zeta/(1-\eta)) + \bar{b}_{ij} l_{ij}^g (M_j^g + l_{2j}^g \zeta/(1-\eta)) + \bar{c}_{ij} l_{ij}^h K_{ij} (M_j^h + l_{2j}^h \zeta/(1-\eta))] + L_i [(\zeta/(1-\eta)) \sum_{j=1}^n (\bar{a}_{ij} l_{ij}^f M_j^f + \bar{b}_{ij} l_{ij}^g M_j^g + \bar{c}_{ij} l_{ij}^h M_j^h K_{ij}) + \bar{I}_i] \} = 2\lambda_i \|\varphi - \phi\|。$$
 所以  $\|\Phi(\varphi) - \Phi(\phi)\| \leq \max\{\max_{1 \leq i \leq n} (2\lambda_i \|\varphi - \phi\|/(\mu_i + \omega_i)), \max_{1 \leq i \leq n} (2\lambda_i \|\varphi - \phi\|)\} = \lambda \|\varphi - \phi\|$ 。因为  $\lambda < 1$ , 所以映射  $\Phi: \Omega^* \rightarrow \Omega^*$  是一个压缩映射。由 Banach 空间的不动点定理可知,  $\Phi$  在  $\Omega^*$  中存在唯一的不动点  $\varphi(t)$ , 即  $\Phi(\varphi) = \varphi$ 。由式 (3) 可知  $\varphi(t)$  满足系统 (1), 故  $\varphi(t)$  是系统 (1) 的唯一概周期解且  $\|\varphi\| \leq \zeta/(1-\eta)$ 。

注 1 文中的方法可以用来研究一些其他具有时滞的 Cohen-Grossberg 神经网络的概周期解问题。

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