

间歇性和滞后效应策略的不连续神经网络同步分析

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[摘要] 研究时滞线性耦合不连续神经网络的同步控制问题。运用李雅普诺夫稳定性理论和微分方程比较定理, 提出一种基于间歇性和滞后效应策略的控制器, 获得时滞线性耦合不连续神经网络的同步准则。最后进行数值模拟, 从而验证所得理论结果的有效性。

[关键词] 不连续神经网络; 间歇性; 滞后效应; 时滞线性耦合; 同步控制

[中图分类号] O 193

Synchronization Analysis of Discontinuous Neural Networks Based on Intermittence and Lag Effect Strategy

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Abstract: In this paper, the synchronization control problem of time-delay linearly coupled discontinuous neural networks was studied. By using Lyapunov stability theory and comparison theorem of differential equations, a controller based on the strategy of intermittence and lag effect was proposed, and the synchronization criteria of time-delay linearly coupled discontinuous neural networks was obtained. Finally, numerical simulation was carried out to verify the validity of the theoretical result.

Keywords: discontinuous neural networks; intermittence; lag effect; time-delay linearly coupled; synchronization control

0 引言

近年来, 不连续激活函数的神经网络被广泛地研究并应用到许多领域, 如冲击机、干摩擦、电源电路等^[1-2]。除了对该类神经网络的平衡解和周期解的稳定性和收敛性^[3-11]进行研究之外, 人们也逐渐关注其同步控制问题。同步在科学与工程领域中的应用越来越多: 文献[12]将分数阶同步系统应用到数字密码学中, 得到一个安全的密钥系统; 文献[13]研究了一类分数阶混沌系统的同步问题, 获得主动控制下的认证加密方案; 文献[14-15]分别将同步应用到基于出生日期的仿射密码、分数阶眼镜王蛇混沌系统中。同步应用到连续系统及混沌系统的成果已经很多, 但关于不连续系统的同步应用却很少。关于复杂网络同步, 人们提出很多方法和实验技术, 如状态反馈控制^[16]、自适应控制^[17-18]、滑模控制^[19]、非线性反馈控制^[20]、模糊系统的控制^[21]、间歇性控制^[22]等, 但是很少有文献同时考虑到间歇性和滞后效应控制。受现有文献的启发, 在 Filippov 解和广义李雅普诺夫稳

[收稿日期] 2019-09-10

[基金项目] 国家自然科学基金项目(61573005); 福建省自然科学基金项目(2018J01417, 2019J01330); 福建省中青年教师教育科研项目(JAT190322)

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定性理论的框架下, 本文结合滞后效应和间歇性的控制策略, 获得不连续神经网络的同步控制准则。

1 问题描述

考虑 N 个耦合节点的神经网络系统, 其状态方程为

$$\dot{\mathbf{x}}_i(t) = -D\mathbf{x}_i(t) + \mathbf{A}\mathbf{f}(\mathbf{x}_i(t)) + \mathbf{B}\mathbf{f}(\mathbf{x}_i(t-\tau)) + \mathbf{I} + \sum_{j=1}^N (c_{ij}\mathbf{F}\mathbf{x}_j(t)) + \sum_{j=1}^N (c_{ij}^{\tau}\mathbf{F}_{\tau}\mathbf{x}_j(t-\tau)), i = 1, 2, \dots, N. \quad (1)$$

其中: $\mathbf{x}_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbf{R}^n$ 为状态向量; $D = \text{diag}\{d_1, d_2, \dots, d_n\}$, $d_i > 0, d_i \in \mathbf{R}$ 为神经元的自我抑制; $\tau > 0$ 为时滞; $\mathbf{A} = (a_{ij})_{n \times n}$ 为反馈连接权重矩阵; $\mathbf{B} = (b_{ij})_{n \times n}$ 为时滞反馈连接权重矩阵; $\mathbf{f}(\mathbf{x}_i(t)) = (f_1(x_{i1}), f_2(x_{i2}), \dots, f_n(x_{in}))^T \in \mathbf{R}^n$ 为不连续非线性激活函数; $\mathbf{I} = (I_1, I_2, \dots, I_n)^T$ 为常数神经元输入向量; $\mathbf{C} = (c_{ij})_{N \times N}$, $\mathbf{C}_{\tau} = (c_{ij}^{\tau})_{N \times N}$ 分别表示非时滞、时滞结构的耦合强度。若节点 i 与节点 j ($i \neq j$) 存在连接, 则 $c_{ij} > 0$, 否则 $c_{ij} = 0$ 。同样, 若节点 i 与节点 j ($i \neq j$) 存在连接, 则

$$c_{ij}^{\tau} > 0, \text{ 否则 } c_{ij}^{\tau} = 0. \text{ 假设 } \mathbf{C} \text{ 与 } \mathbf{C}_{\tau} \text{ 为扩散矩阵, 满足 } c_{ii} = -\sum_{j=1, j \neq i}^N c_{ij}, c_{ii}^{\tau} = -\sum_{j=1, j \neq i}^N c_{ij}^{\tau}.$$

耦合网络的初始条件为 $x_{ij}(s) = \bar{w}_{ij}(s) \in \Psi([- \tau, 0], \mathbf{R})$, 其中 $\Psi([- \tau, 0], \mathbf{R})$ 表示从 $[- \tau, 0]$ 到 \mathbf{R} 的连续函数的全体。

网络 (1) 的孤立节点可表示为

$$\dot{\mathbf{x}}(t) = -D\mathbf{x}(t) + \mathbf{A}\mathbf{f}(\mathbf{x}(t)) + \mathbf{B}\mathbf{f}(\mathbf{x}(t-\tau)) + \mathbf{I}, \quad (2)$$

其中: $\mathbf{x}(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ 为神经网络的状态向量; $\mathbf{f}(\mathbf{x}(t)) = (f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t)))^T$ 为神经元的输出, 是不连续函数向量。

关于不连续函数和连接矩阵参数, 本文作如下假设。

假设 1 任意的不连续函数 f_i 只在可数的孤立点集 $\{\vartheta_k^i\}$ 上不连续, f_i 在孤立点处的左右极限 $f_i^-(\vartheta_k^i), f_i^+(\vartheta_k^i)$ 均存在 (为有限的数)。此外, f_i 在 \mathbf{R} 的每一个闭区间至多只有有限个跳跃间断点, $i = 1, 2, \dots, n$ 。

假设 2 $\forall i = 1, 2, \dots, n$, 存在非负常数 L_i, N_i , 使得 $\sup |\xi_i - \eta_i| \leq L_i |u - v| + N_i$, 其中 $u, v \in \mathbf{R}$, $\xi_i \in K[f_i(u)], \eta_i \in K[f_i(v)], K[f_i(s)] = [\min\{f_i^-(s), f_i^+(s)\}, \max\{f_i^-(s), f_i^+(s)\}]$, $s \in \mathbf{R}$ 。

假设 3 耦合矩阵 $\|\mathbf{F}\| = r > 0$, ρ_{\min} 为矩阵 $(\mathbf{F}^T + \mathbf{F})/2$ 的最小特征值。

假设 4 \mathbf{C}_{τ} 的对角元素满足 $|c_{ii}^{\tau}| \leq g$ 。

引理 1^[3] 若假设 1、假设 2 满足, 则非连续系统 (2) 任意初值问题至少有一个定义在 $[0, +\infty]$ 上的解 $[x, \gamma]$ 。

根据文献 [19] 中初值问题的定义和引理 2.10, 系统 (1) 至少有一个解 $[x_i(t), \gamma_i(t)]$ 。

关于神经网络 (1) 的同步定义如下。

定义 1 设 $[x_i(t), \gamma_i(t)]$ ($i = 1, 2, \dots, N$) 是系统 (1) 的一个解, $[x^*(t), \gamma^*(t)]$ 是具有相应初值条件系统 (2) 的一个解。若 $\lim_{t \rightarrow \infty} \|x_i(t) - x^*(t)\| = 0, i = 1, 2, \dots, N$, 则线性耦合的不连续神经网络达到同步。

控制神经网络 (1) 的描述如下:

$$\dot{\mathbf{x}}_i(t) = -D\mathbf{x}_i(t) + \mathbf{A}\mathbf{f}(\mathbf{x}_i(t)) + \mathbf{B}\mathbf{f}(\mathbf{x}_i(t-\tau)) + \mathbf{I} + \sum_{j=1}^N (c_{ij}\mathbf{F}\mathbf{x}_j(t)) + \sum_{j=1}^N (c_{ij}^{\tau}\mathbf{F}_{\tau}\mathbf{x}_j(t-\tau)) + u_i(t), i = 1, 2, \dots, N, \quad (3)$$

其中 $u_i(t)$ 是后文中将要设计的控制输入。

定义误差向量 $\mathbf{e}_i(t) = x_i(t) - x^*(t)$, $\tilde{\boldsymbol{\gamma}}_i = \boldsymbol{\gamma}_i(t) - \boldsymbol{\gamma}^*(t)$, $i = 1, 2, \dots, N$, 得到如下误差系统

$$\begin{aligned} \dot{\mathbf{e}}_i(t) = & -D\mathbf{e}_i(t) + \mathbf{A}\tilde{\boldsymbol{\gamma}}_i(t) + \mathbf{B}\tilde{\boldsymbol{\gamma}}_i(t - \tau) + \sum_{j=1}^N (c_{ij}\boldsymbol{\Gamma}\mathbf{e}_j(t)) + \\ & \sum_{j=1}^N (c_{ij}^{\tau}\boldsymbol{\Gamma}_{\tau}\mathbf{e}_j(t - \tau)) + u_i(t), i = 1, 2, \dots, N. \end{aligned} \tag{4}$$

设计基于间歇性和滞后效应策略的控制器

$$u_i(t) = -k(t)\mathbf{e}_i(t) - \eta_i \text{sign}(\mathbf{e}_i(t)) - \xi_i \|\mathbf{e}(t - \tau)\|_2^2 \mathbf{e}_i(t), \tag{5}$$

使得神经网络 (3) 同步于 (2)。其中: $k(t) = \begin{cases} k, & nT \leq t < (n + \theta)T \\ 0, & (n + \theta)T \leq t < (n + 1)T \end{cases}$, $0 < \theta < 1, n = 0, 1, 2, \dots, k > 0$; $\eta_i, \xi_i \in \mathbf{R}$ 为反馈参数; $\mathbf{e}(t - \tau) = (e_1(t - \tau)^T, e_2(t - \tau)^T, \dots, e_N(t - \tau)^T)^T$ 。

2 主要结论

定理 1 假设 1 ~ 假设 4 成立, 若控制器 (5) 满足: $\eta_i > \alpha_k + \beta_k$, $\sum_{i=1}^N (\xi_i \mathbf{e}_i^T(t) \mathbf{e}_i(t)) > 1$, $\bar{k}(t) > a + r\lambda - d_{\min}$, 则此控制器能实现网络 (3) 与 (2) 的同步。其中: $a = \lambda_{\max}(|\bar{\mathbf{A}}| + |\bar{\mathbf{B}}| |\bar{\mathbf{B}}|^T / 2 + N^2 g^2 \boldsymbol{\Gamma}_{\tau}^T \boldsymbol{\Gamma}_{\tau} / 2)$; $d_{\min} = \min\{d_i\}$; $\lambda = \lambda_{\max}((\hat{\mathbf{C}}^T + \hat{\mathbf{C}}) / 2)$; $|\bar{\mathbf{A}}| = |\mathbf{A}| \mathbf{L}$, $|\mathbf{A}| = (|a_{ij}|)_{n \times n}$, $\mathbf{L} = \text{diag}(L_1, L_2, \dots, L_n)$; $|\bar{\mathbf{B}}| = |\mathbf{B}| \mathbf{L}$, $|\mathbf{B}| = (|b_{ij}|)_{n \times n}$; $\alpha_k = \sum_{p=1}^n |a_{kp}| N_p$; $\beta_k = \sum_{p=1}^n |b_{kp}| N_p$; $\bar{k}(t) = \int_0^T k(s) \text{d}s / T = \int_0^{\theta T} k \text{d}s / T = k\theta$ 。

证明 考虑李雅普诺夫函数 $V(x) = \sum_{i=1}^N (\mathbf{e}_i^T(t) \mathbf{e}_i(t)) / 2$ 。沿着误差系统 (4) 对 $V(t)$ 关于 t 求导, 可得 $\dot{V}(x) = \sum_{i=1}^N (\mathbf{e}_i^T(t) \dot{\mathbf{e}}_i(t)) = \sum_{i=1}^N \mathbf{e}_i^T(t) [-D\mathbf{e}_i(t) + \mathbf{A}\tilde{\boldsymbol{\gamma}}_i(t) + \mathbf{B}\tilde{\boldsymbol{\gamma}}_i(t - \tau) + \sum_{j=1}^N (c_{ij}\boldsymbol{\Gamma}\mathbf{e}_j(t)) + \sum_{j=1}^N (c_{ij}^{\tau}\boldsymbol{\Gamma}_{\tau}\mathbf{e}_j(t - \tau)) - k(t)\mathbf{e}_i(t) - \eta_i \text{sign}(\mathbf{e}_i(t)) - \xi_i \|\mathbf{e}(t - \tau)\|_2^2 \mathbf{e}_i(t)]$ 。

又 $-\sum_{i=1}^N (\mathbf{e}_i^T(t) D\mathbf{e}_i(t)) \leq -d_{\min} \sum_{i=1}^N (\mathbf{e}_i^T(t) \mathbf{e}_i(t))$, $\mathbf{e}_i^T(t) \mathbf{A}\tilde{\boldsymbol{\gamma}}_i(t) = \sum_{k=1}^n \sum_{p=1}^n (\mathbf{e}_{ik}^T(t) a_{kp} \tilde{\boldsymbol{\gamma}}_{ip}(t)) \leq \sum_{k=1}^n \sum_{p=1}^n [|\mathbf{e}_{ik}^T(t)| |a_{kp}| (L_p |\mathbf{e}_{ip}(t)| + N_p)] = \sum_{k=1}^n \sum_{p=1}^n [|\mathbf{e}_{ik}^T(t)| |a_{kp}| L_p |\mathbf{e}_{ip}(t)|] + \sum_{k=1}^n \sum_{p=1}^n (|\mathbf{e}_{ik}^T(t)| |a_{kp}| N_p) \leq |\mathbf{e}_i(t)|^T |\bar{\mathbf{A}}| |\mathbf{e}_i(t)| + \sum_{k=1}^n (\alpha_k |\mathbf{e}_{ik}(t)|)$, $\mathbf{e}_i^T(t) \mathbf{B}\tilde{\boldsymbol{\gamma}}_i(t - \tau) = \sum_{k=1}^n \sum_{p=1}^n (\mathbf{e}_{ik}^T(t) b_{kp} \tilde{\boldsymbol{\gamma}}_{ip}(t - \tau)) \leq \sum_{k=1}^n \sum_{p=1}^n [|\mathbf{e}_{ik}^T(t)| |b_{kp}| (L_p |\mathbf{e}_{ip}(t - \tau)| + N_p)] = \sum_{k=1}^n \sum_{p=1}^n (|\mathbf{e}_{ik}^T(t)| |b_{kp}| L_p |\mathbf{e}_{ip}(t - \tau)|) + \sum_{k=1}^n \sum_{p=1}^n (|\mathbf{e}_{ik}^T(t)| |b_{kp}| N_p) \leq |\mathbf{e}_i(t)|^T |\bar{\mathbf{B}}| |\mathbf{e}_i(t - \tau)| + \sum_{k=1}^n (\beta_k |\mathbf{e}_{ik}(t)|) \leq |\mathbf{e}_i(t)|^T |\bar{\mathbf{B}}| |\bar{\mathbf{B}}|^T |\mathbf{e}_i(t)| / 2 + |\mathbf{e}_i(t - \tau)|^T |\mathbf{e}_i(t - \tau)| / 2 + \sum_{k=1}^n (\beta_k |\mathbf{e}_{ik}(t)|)$, $\sum_{i=1}^N \sum_{j=1}^N (c_{ij} \mathbf{e}_i^T(t) \boldsymbol{\Gamma} \mathbf{e}_j(t)) = \sum_{i=1}^N \sum_{j=1, j \neq i}^N (c_{ij} \mathbf{e}_i^T(t) \boldsymbol{\Gamma} \mathbf{e}_j(t)) + \sum_{i=1}^N [c_{ii} \mathbf{e}_i^T(t) [(\boldsymbol{\Gamma}^T + \boldsymbol{\Gamma}) / 2] \mathbf{e}_i(t)] \leq \sum_{i=1}^N \sum_{j=1, j \neq i}^N (rc_{ij} \|\mathbf{e}_i(t)\|_2 \|\mathbf{e}_j(t)\|_2) + \sum_{i=1}^N (c_{ii} \rho_{\min} \mathbf{e}_i^T(t) \mathbf{e}_i(t)) = \mathbf{e}^T(t) r [(\hat{\mathbf{C}}^T + \hat{\mathbf{C}}) / 2] \mathbf{e}(t) \leq r\lambda \sum_{i=1}^N (\mathbf{e}_i^T(t) \mathbf{e}_i(t))$, 其中 $\hat{\mathbf{C}}$ 为 \mathbf{C} 的修正矩阵, 即用 $(\rho_{\min} / r) c_{ii}$ 代替 \mathbf{C} 中的 c_{ii} , $\mathbf{e}(t) = (\|\mathbf{e}_1(t)\|_2, \|\mathbf{e}_2(t)\|_2, \dots, \|\mathbf{e}_N(t)\|_2)^T$ 。

由 $c_{ii}^\tau = -\sum_{j=1, j \neq i}^N c_{ij}^\tau$, $c_{ij}^\tau \geq 0$ 知, $|c_{ij}^\tau| \leq |c_{ii}^\tau|$ 。从而, 由假设 4 可得 $\sum_{i=1}^N \sum_{j=1}^N (\mathbf{e}_i^\mathrm{T}(t) c_{ij}^\tau \mathbf{I}_\tau \mathbf{e}_j(t-\tau)) \leq \sum_{i=1}^N \sum_{j=1}^N [(c_{ij}^\tau)^2 \mathbf{e}_i^\mathrm{T}(t) \mathbf{I}_\tau R \mathbf{I}_\tau^\mathrm{T} \mathbf{e}_i(t)]/2 + \sum_{i=1}^N \sum_{j=1}^N [\mathbf{e}_j^\mathrm{T}(t-\tau) R^{-1} \mathbf{e}_j(t-\tau)]/2 \leq N g^2 \sum_{i=1}^N [\mathbf{e}_i^\mathrm{T}(t) \mathbf{I}_\tau R \mathbf{I}_\tau^\mathrm{T} \mathbf{e}_i(t)]/2 + \sum_{j=1}^N [\mathbf{e}_j^\mathrm{T}(t-\tau) R^{-1} \mathbf{e}_j(t-\tau)]/2 = N^2 g^2 \sum_{i=1}^N [\mathbf{e}_i^\mathrm{T}(t) \mathbf{I}_\tau \mathbf{I}_\tau^\mathrm{T} \mathbf{e}_i(t)]/2 + \sum_{j=1}^N [\mathbf{e}_j^\mathrm{T}(t-\tau) \mathbf{e}_j(t-\tau)]/2$, 其中 $R = NI > 0$ 。于是, $\dot{V} \leq \sum_{i=1}^N \{[-d_{\min} + \lambda_{\max}(|\bar{\mathbf{A}}| + |\bar{\mathbf{B}}| |\bar{\mathbf{B}}|^\mathrm{T}/2 + N^2 g^2 \mathbf{I}_\tau \mathbf{I}_\tau^\mathrm{T}/2) + r\lambda] \mathbf{e}_i^\mathrm{T}(t) \mathbf{e}_i(t)\} + \sum_{i=1}^N \sum_{k=1}^n [(\alpha_k + \beta_k) |\mathbf{e}_{ik}(t)|] + \sum_{j=1}^N [\mathbf{e}_j^\mathrm{T}(t-\tau) \mathbf{e}_j(t-\tau)] + \sum_{i=1}^N \{ \mathbf{e}_i^\mathrm{T}(t) [-k(t) \mathbf{e}_i(t) - \eta_i \text{sign}(\mathbf{e}_i(t))] - \xi_i \|\mathbf{e}(t-\tau)\|_2^2 \mathbf{e}_i(t) \} = \sum_{i=1}^N \{[-d_{\min} + a + r\lambda - k(t)] \mathbf{e}_i^\mathrm{T}(t) \mathbf{e}_i(t)\} + \sum_{i=1}^N \sum_{k=1}^n \{[(\alpha_k + \beta_k) - \eta_i] |\mathbf{e}_{ik}(t)|\} + \|\mathbf{e}(t-\tau)\|_2^2 [1 - \sum_{i=1}^N \xi_i \mathbf{e}_i^\mathrm{T}(t) \mathbf{e}_i(t)]$ 。又 $\eta_i > \alpha_k + \beta_k$, $\sum_{i=1}^N (\xi_i \mathbf{e}_i^\mathrm{T}(t) \mathbf{e}_i(t)) > 1$, 从而 $\dot{V} \leq -2[k(t) + d_{\min} - a - r\lambda]V$ 。

设 $\dot{U}(t) = -2[k(t) + d_{\min} - a - r\lambda]U(t)$, $U(0) = V(0)$, $t = mT + t_1$ ($0 \leq t_1 < T$), 由微分方程比较定理^[23]知, $V(t) \leq U(t) = U(0) \exp\{-2 \int_0^t [k(s) + d_{\min} - a - r\lambda] ds\} = U(0) \exp\{-2 \int_0^{mT} [k(s) + d_{\min} - a - r\lambda] ds\} \times \exp\{-2 \int_{mT}^{mT+t_1} [k(s) + d_{\min} - a - r\lambda] ds\} = U(0) \exp\{-2m \int_0^{\theta T} [k(s) + d_{\min} - a - r\lambda] ds\} \times \exp\{-2 \int_{mT}^{mT+t_1} [k(s) + d_{\min} - a - r\lambda] ds\} = U(0) \exp\{-2m[\int_0^{\theta T} (k + d_{\min} - a - r\lambda) ds + \int_{\theta T}^T (d_{\min} - a - r\lambda) ds]\} \times \exp\{-2 \int_{mT}^{mT+t_1} [k(s) + d_{\min} - a - r\lambda] ds\} \leq U(0) \exp\{-2m[\int_0^{\theta T} (k + d_{\min} - a - r\lambda) ds + \int_{\theta T}^T (d_{\min} - a - r\lambda) ds]\} \times \exp\{-2 \int_{mT}^{mT+t_1} (d_{\min} - a - r\lambda) ds\} \leq U(0) \exp\{-2m[(k + d_{\min} - a - r\lambda)\theta T + (d_{\min} - a - r\lambda)(1-\theta)T]\} \times \exp[-2(d_{\min} - a - r\lambda)t_1] \leq U(0) \exp[-2mT(k\theta + d_{\min} - a - r\lambda) + 2(-d_{\min} + a + r\lambda)t_1]$ 。

又 $mT = t - t_1$, 从而 $V(t) \leq U(0) \exp[-2(t-t_1)(k\theta + d_{\min} - a - r\lambda) + 2(-d_{\min} + a + r\lambda)t_1] = U(0) \exp[-2t(k\theta + d_{\min} - a - r\lambda)] \times \exp(2k\theta t_1) \leq U(0) \exp[-2t(k\theta + d_{\min} - a - r\lambda)] \times \exp(2k\theta T)$ 。因此, $\|V(t)\| \leq M \exp[-2t(k\theta + d_{\min} - a - r\lambda)] = M \exp\{-2t[\bar{k}(t) + d_{\min} - a - r\lambda]\}$, 其中 $M = U(0) \exp(2k\theta T)$ 。故 $\lim_{t \rightarrow \infty} V(t) = 0$, 从而 $\lim_{t \rightarrow \infty} \mathbf{e}(t) = 0$, 系统 (3) 与 (2) 同步。

文献 [18] 利用广义李雅普诺夫方法, 设计状态反馈控制器 $u_i(t) = -k_i \mathbf{e}_i(t) - \eta_i \text{sign}(\mathbf{e}_i(t))$, 实现了不连续激活的线性耦合时滞神经网络的同步。在文献 [18] 的基础上, 本文给出了间歇性控制策略, 并运用李雅普诺夫稳定性理论和微分方程比较定理加以证明, 所得结果具有更好的适用性。

3 数值模拟

为了验证以上的理论分析, 本节给出一组参数来进行数值模拟。

例 1 考虑 3 个耦合节点, 每个节点是 1 维的神经网络。网络的孤立节点方程为

$$\dot{x}(t) = -x(t) + 1.25f(x(t)) - 2.3f(x(t-\tau)), \quad (6)$$

其中: $\tau=2$; 激活函数 $f(x(t)) = \begin{cases} \tanh(x(t)) + 0.05, & x(t) \geq 0 \\ \tanh(x(t)) - 0.03, & x(t) < 0 \end{cases}$; 初值 $x(t) = 1/3, t \leq 0$ 。图 1 为

网络 (6) 的轨迹图。

显然, 网络 (6) 满足假设 1 与假设 2, $L = 1, N = 0.08$ 。3 个耦合节点的时滞神经控制网络为

$$\dot{\mathbf{x}}_i(t) = -\mathbf{x}_i(t) + 1.25f(\mathbf{x}_i(t)) - 2.3f(\mathbf{x}_i(t-\tau)) + 1.2\left(\sum_{j=1}^3 c_{ij}\mathbf{x}_j(t)\right) + \sum_{j=1}^3 c_{ij}^{\tau}\mathbf{x}_j(t-\tau) - k(t)\mathbf{e}_i(t) - \eta_i\text{sign}(\mathbf{e}_i(t)) - \xi_i\|\mathbf{e}(t-\tau)\|_2^2\mathbf{e}_i(t), i = 1, 2, 3.$$

(7)

其中: $\mathbf{C} = (c_{ij})_{3 \times 3} = \begin{pmatrix} -0.1 & 0 & 0.1 \\ 0 & -0.2 & 0.2 \\ 0 & 0.1 & -0.1 \end{pmatrix}$; $\mathbf{C}_{\tau} = (c_{ij}^{\tau})_{3 \times 3} = \begin{pmatrix} -0.2 & 0.1 & 0.1 \\ 0.2 & -0.2 & 0 \\ 0.2 & 0.1 & -0.3 \end{pmatrix}$; 初值 $x_1(t) = 3/2$, $x_2(t) = 2/3$, $x_3(t) = -1/2$, $t \leq 0$ 。容易算出, $g = 0.3, a = 4.3, r = 1.2, \lambda = 0.0219, d_{\min} = 1$ 。取 $\theta = 1/2$, $T = 0.005$, $k = 6.8$, $\eta_i = 0.3$, $\xi_i = 100$, $i = 1, 2, 3$, 得到误差图 (见图 2)。由图 2 可以看出系统 (7) 与 (6) 同步。

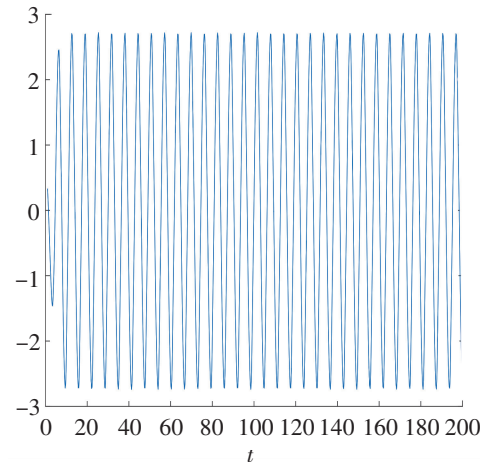


图 1 网络(6)的轨迹图
Fig.1 Trajectory of system(6)

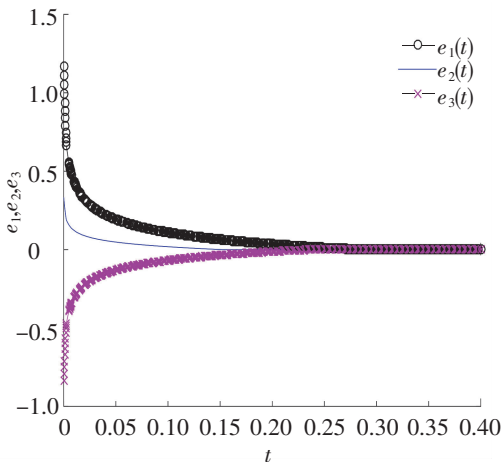


图 2 系统(7)与(6)的误差轨迹图
Fig.2 Error trajectory of system(7) and (6)

4 结论

本文提出了一种结合间歇性和滞后效应的控制策略, 探讨了一类线性耦合不连续神经网络的同步问题, 所得的结果是文献 [18] 的补充与推广, 具有很好的适用性。数值仿真验证了理论结果的有效性。针对文中这类线性耦合不连续神经网络, 是否有更好的同步控制策略, 可以作为今后的一个研究方向。

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